Dynamics of an Insulated Rail Joint

Principles of Machine Design with FEM Applications
Exam project

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Project description: Insulated Rail Joint

Function: Ensures structural continuity of Continuous welded rails maintaining electrical isolation for signalling circuits

Components:

- Rails (separated by a standard gap)
- Fishplates with insulating liners
- Insulating End-Post: focus of the failure simulation

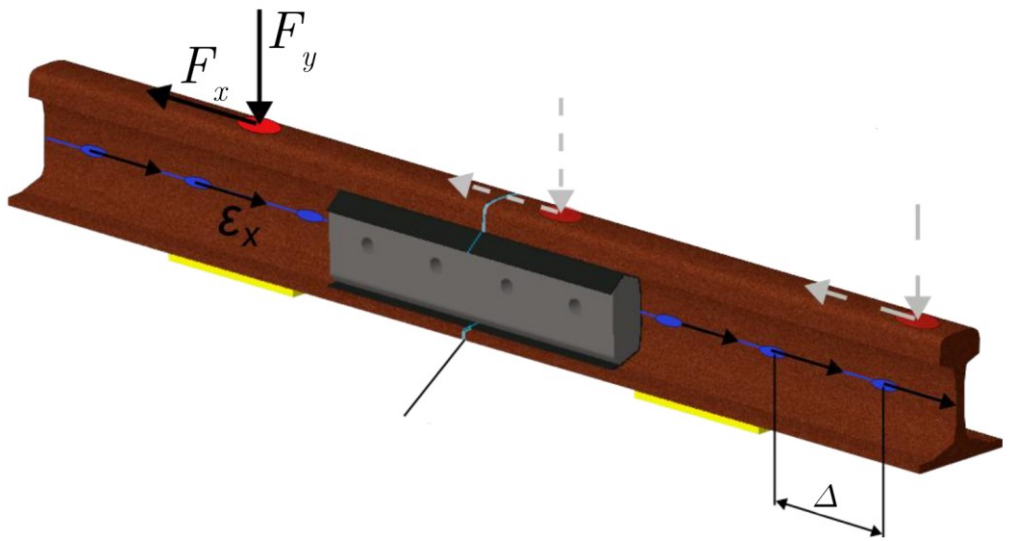
Boundary conditions:

- Hertzian train load travelling at v = 8 m/s (Fz = 150 kN, Fx = 30 kN)

$$p(x,y) = p_0\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}} \qquad p_0 = \frac{3F_z}{2\pi ab}$$

- Elastic support

Direction	Symbol	Coefficient [MN/m]
Longitudinal (x)	k_x	10
Transverse (y)	k_y	15
Vertical (z)	k_z	100



Probes:

- Spatial resolution: at least 100 mm
- Longitudinal strain ε_x

Task:

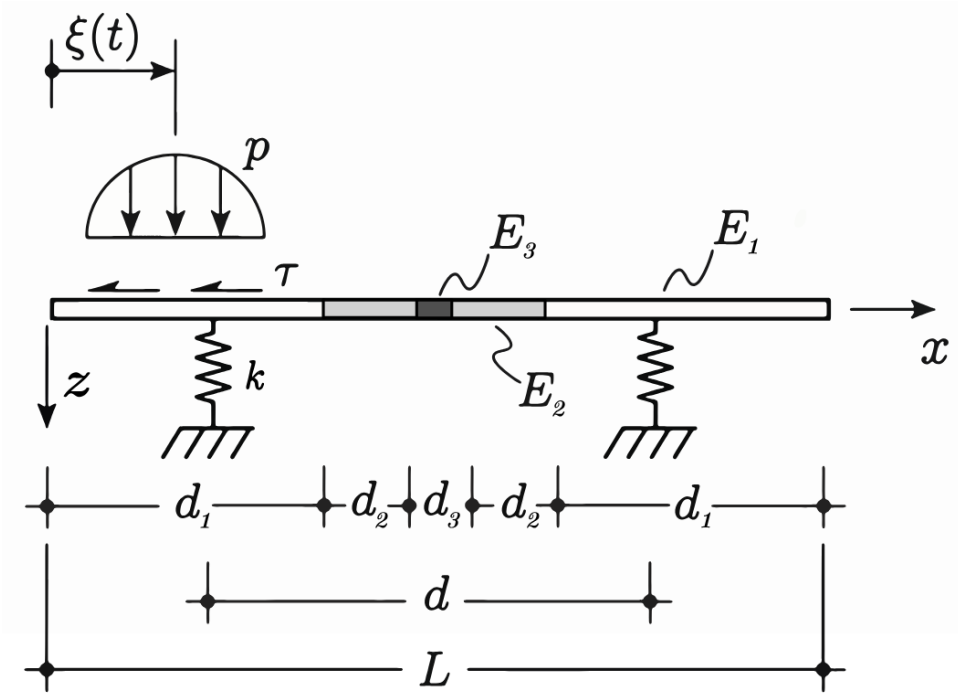
- Reconstruct the strain wave $\varepsilon_x(x,t)$
- Simulate failure of insulating end post

Analytical model

$$\begin{cases} m(x)\frac{\partial^2}{\partial t^2}w(x,t) + \frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2}{\partial x^2}(x,t)\right] = p(x-vt) - k_z(x)w(x,t) \\ m(x)\frac{\partial^2}{\partial t^2}u(x,t) - \frac{\partial}{\partial x}\left[EA(x)\frac{\partial}{\partial x}w(x,t)\right] = \tau(x-vt) - k_x(x)u(x,t) \\ + \textbf{B.C.} \end{cases}$$

$$\begin{cases} w(x_i^-) = w(x_i^+) & \text{(Continuity of displacement)} \\ \frac{\partial w}{\partial x}(x_i^-) = \frac{\partial w}{\partial x}(x_i^+) & \text{(Continuity of slope)} \\ EI(x_i^-)\frac{\partial^2 w}{\partial x^2}(x_i^-) = EI(x_i^+)\frac{\partial^2 w}{\partial x^2}(x_i^+) & \text{(Equilibrium of bending moment)} \\ \frac{\partial}{\partial x}\left(EI(\frac{\partial^2 w}{\partial x^2}\right)_{x_i^-} = \frac{\partial}{\partial x}\left(EI_2\frac{\partial^2 w}{\partial x^2}\right)_{x_i^+} & \text{(Equilibrium of shear force)} \end{cases}$$

$$\begin{cases} u(x_i^-) = u(x_i^+) & \text{(Continuity of axial displacement)} \\ EA(x_i^-)\frac{\partial u}{\partial x}(x_i^-) = EA(x_i^+)\frac{\partial u}{\partial x}(x_i^+) & \text{(Equilibrium of axial force)} \end{cases}$$



The problem:
The rail is a piecewise continuous structure. Solving the non-homogeneous equations independently across three distinct zones with varying properties (EI, EA, m) is exceptionally cumbersome

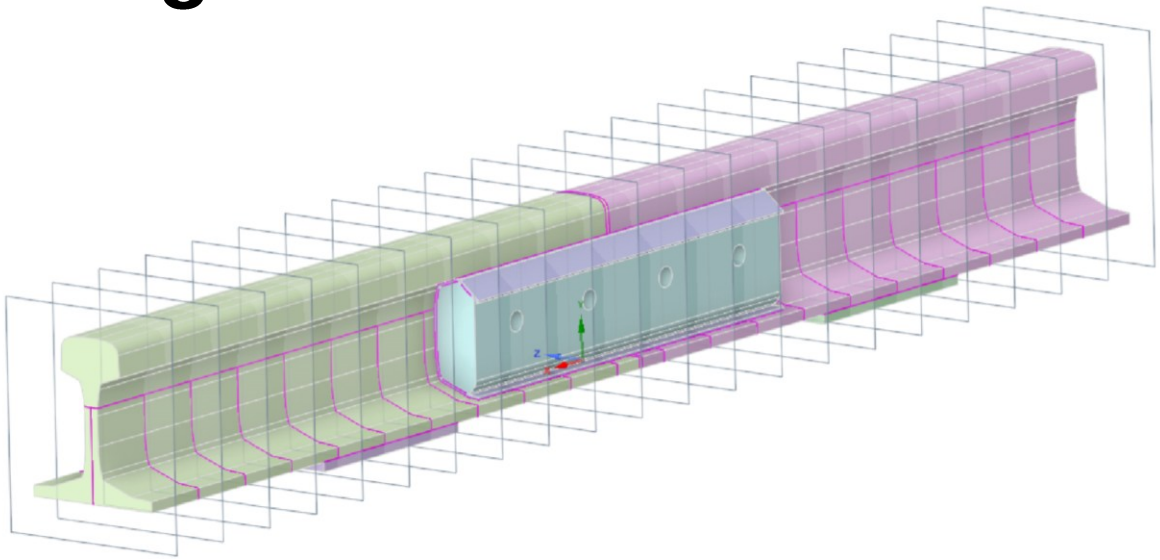
The solution:
Finite element method

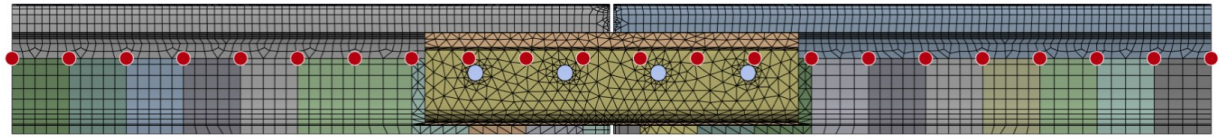
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Geometry partitioning and meshing

Geometry preparation in Ansys SpaceClaim

- Partitioned the model into **multiple bodies** to force exact **node alignment** with the **22** probes (**76.43 mm** interval)
- Separated prismatic sections to generate to force the mesher to produce higher quality elements
- Split faces that will become the elastic support





Geometry partitioning and meshing

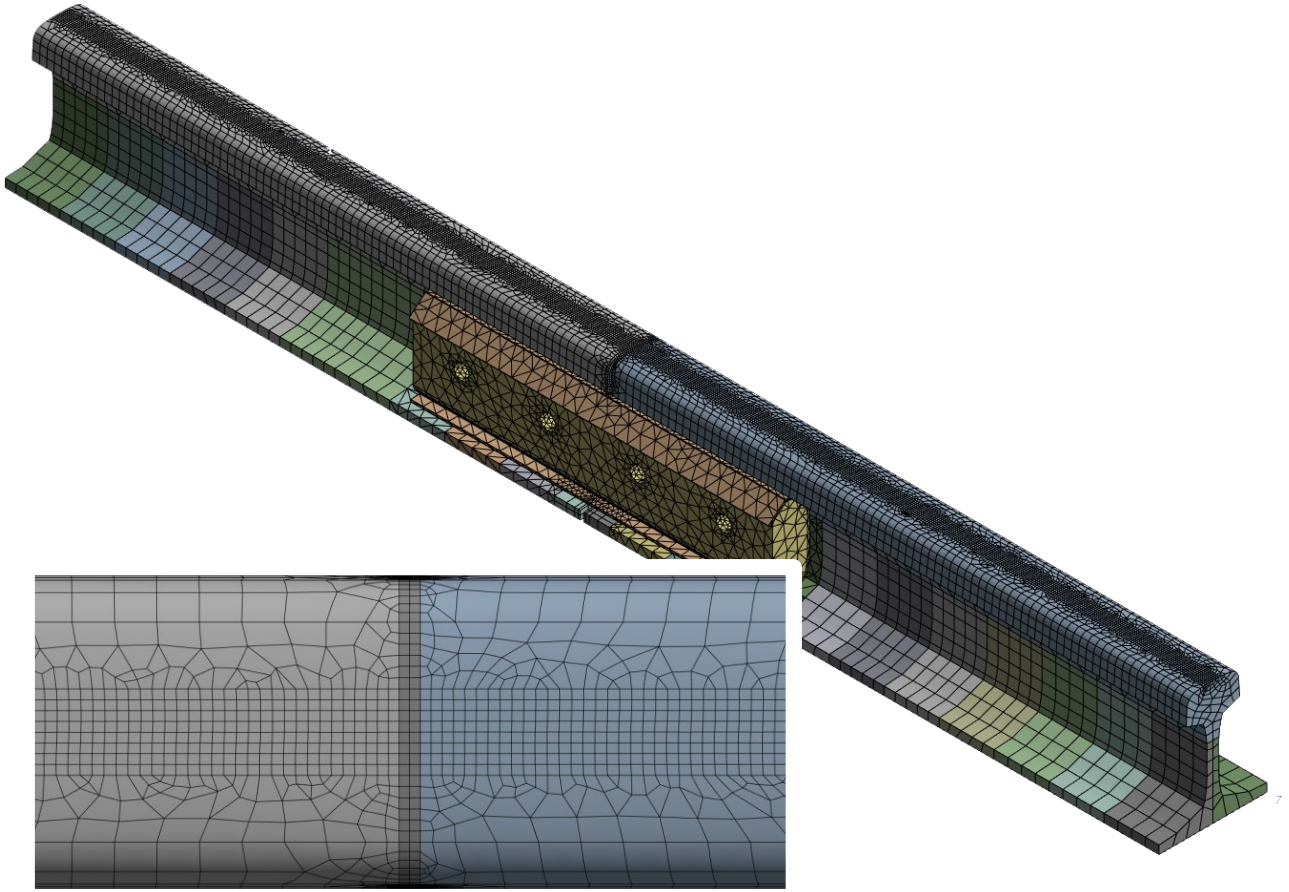
Meshing in Ansys Mechanical Workbench

- **Hex-dominant** method for upper rail and end-post
- **Tetrahedra** mesh for fish-plate

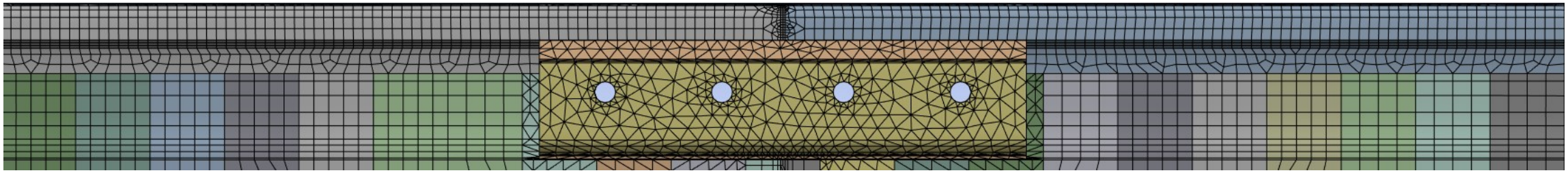
- **Local Refinement**: **2.5 mm** element size
  - at **the load application zone** to model accurately the **10 mm** minor axis of the Hertzian contact ellipse
  - at the **end-post** to avoid **artificial stiffness**
  - Additional local refinements for **bolt holes**

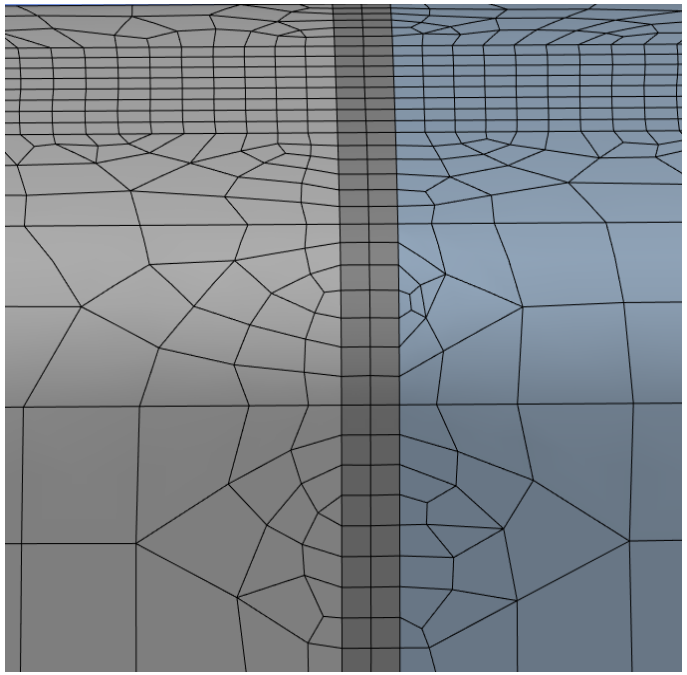
- **Optimization:** Coarse **16 mm** Swept mesh on the rail base to reduce element count

- **Iterative process:** balancing fidelity over computational resources

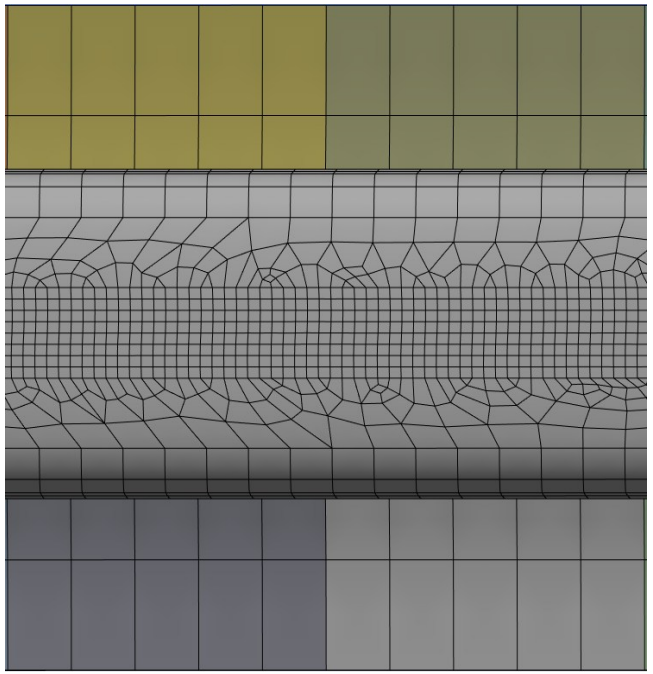


Meshing

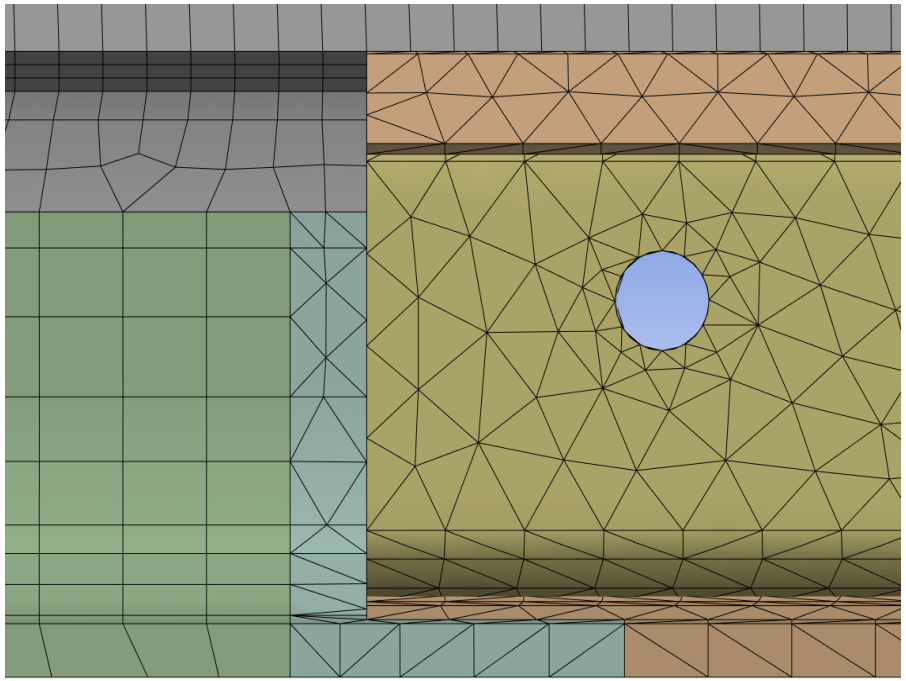




End-post



Load application patch

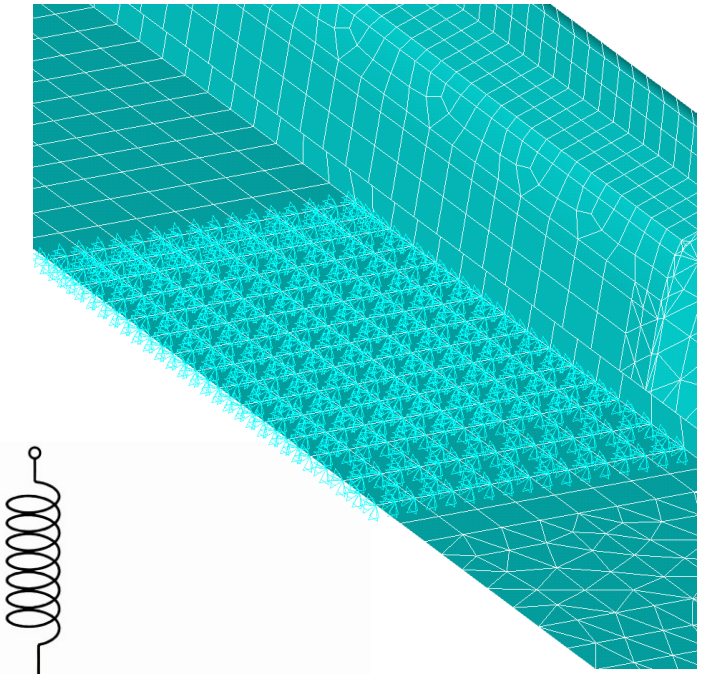


Fishplate and hole treatment

APDL Modelling

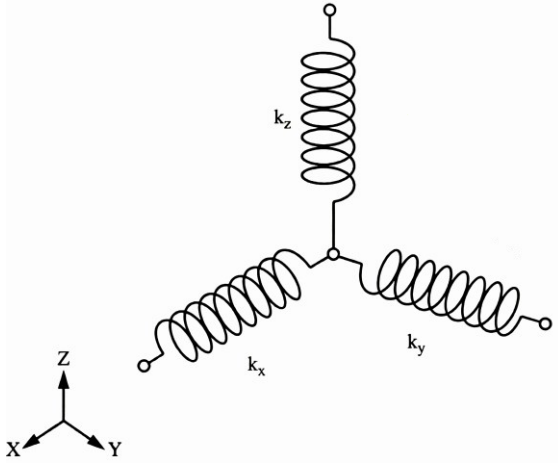
ANSYS Mechanical Workbench does not support surface loads varying simultaneously in space and in time:

$$p = p(x, y, t)$$



Elastic Foundation: COMBIN14 spring elements to simulated distributed nodal stiffness

$$k_{\text{node},i} = \frac{k_i}{N_{\text{nodes}}} \qquad i = x, y, z$$

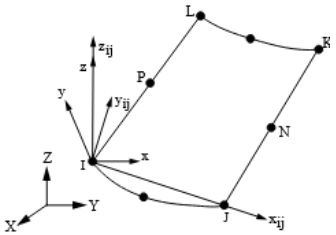


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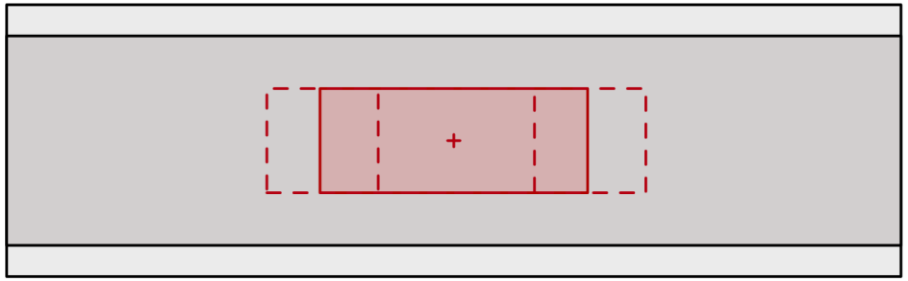
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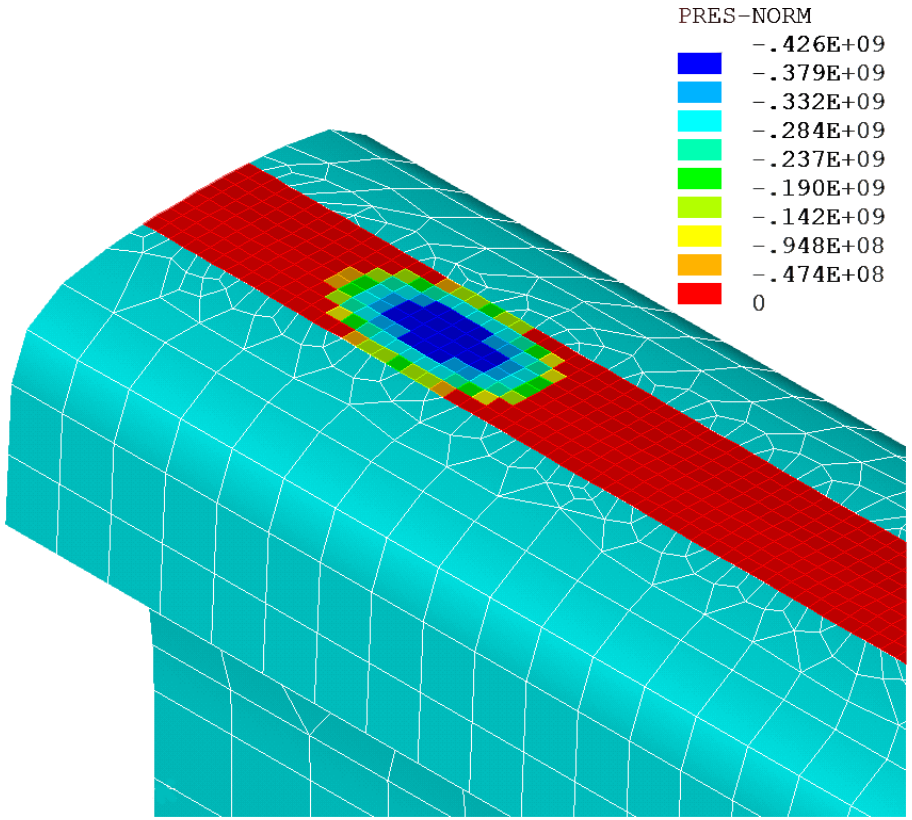
APDL Modelling

Moving load application: SURF154 surface effect elements generated on the rail head



Spatial Pre-filtering: instead of calculating null pressures outside of the contact zones, the code loops over a restricted bounding box cantered on the load, drastically reducing computation





Transient analysis

(1) Linear analysis

- **Low strain** peaks
- $\sigma < \sigma^Y$
- Linear constitutive law $\{\sigma\} = [C]\{\varepsilon\}$

(2) Time step

- Fixed time step **Δt = 0.2 ms** $\Rightarrow f_N = 2500Hz$
- The pressure print falls **25 times** within the contact ellipse reducing **the introduction of artifacts**

(3) Preconditioned Conjugate Gradient solver vs Direct Solver

- **PCG**: iterative approach to converge on a solution, making it significantly more memory-efficient for models with many degrees of freedom

- **Sparse Direct Solver:** instead of approximating the solution through successive iterations, the solver factorizes the stiffness matrix [K] into lower and upper triangular matrices

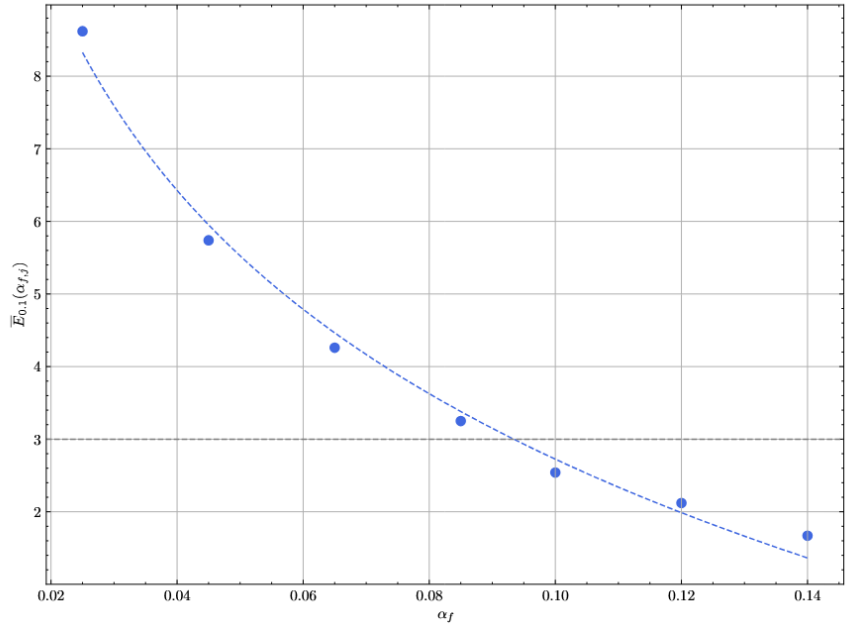
Transient analysis

(4) HHT time integration method

- Suppresses un-physical high-frequency numerical noise
- **Energy dissipation method**

$$D_{\text{ratio}} = \frac{E_{\text{diss}}}{E_{\text{mech}}} = \frac{|E_{\text{mech}} - W_{\text{ext}}|}{E_{\text{mech}}}$$

- **Parametric study**



$$[M]\{\ddot{u}(t)\} + [K]\{u(t)\} = \{F^{\mathrm{a}}(t)\}$$

$$[M]\{\ddot{u}_{n+1}\} + (1-\alpha_f)[K]\{u_{n+1}\} + \alpha_f[K]\{u_n\} = (1-\alpha_f)[F_{n+1}] + \alpha_f[F_n]$$

$$\{u_{n+1}\} = \{u_n\} + \Delta t\{\dot{u}_n\} + \Delta t^2\left[\left(\frac{1}{2} - \beta\right)\{\ddot{u}_n\} + \beta\{\ddot{u}_{n+1}\}\right]$$

$$\{\dot{u}_{n+1}\} = \{\dot{u}_n\} + \Delta t[(1-\gamma)\{\ddot{u}\}_n + \gamma\{\ddot{u}_{n+1}\}]$$

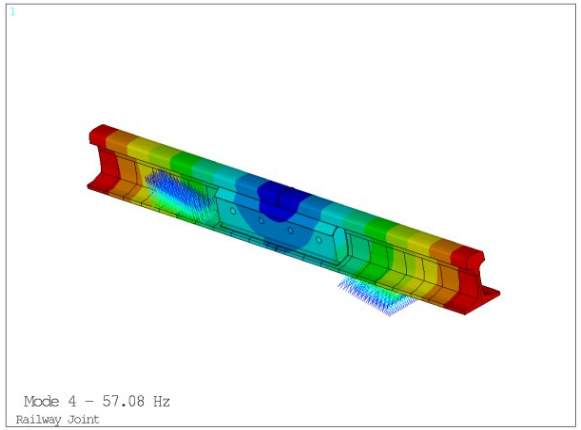
$$\gamma = \frac{1}{2} + \alpha_f \quad \text{and} \quad \beta = \frac{(1+\alpha_f^2)}{4}$$

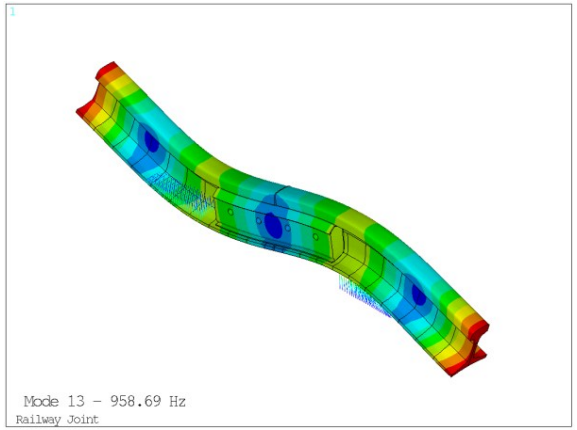
Modal analysis

$$[M]\{\ddot{u}\} + [K]\{u\} = \{0\}$$ Assuming **harmonic motion**: $$([K] - \omega_i^2[M])\{\varphi_i\} = \{0\}$$

- Extracted the first **25 natural frequencies** and **associated mode shapes**
- **Nyquist Limit:** frequencies above **2.5 kHz** were ignored due to the sampling rate

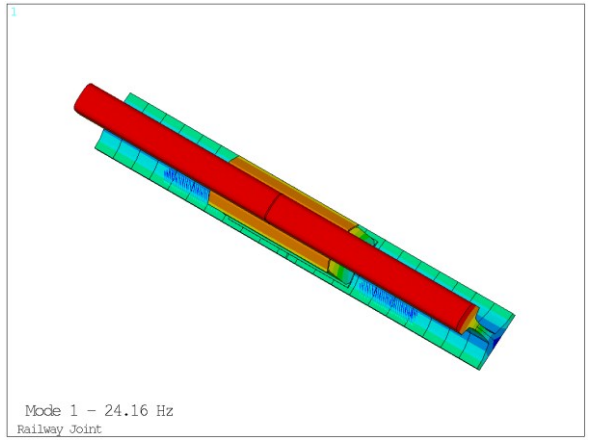
- **Semi-rigid mode shapes:** up to the 4th mode
- **Local Modes:** subsequent natural frequencies are associated to local modes
- **Mesh independence:** sub 1Hz variation in natural frequency
- **Damage effect:** no significant variation in natural frequencies is introduced by the damaged end-post

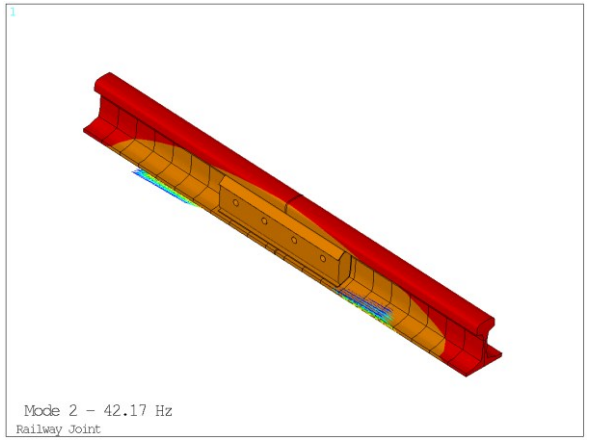


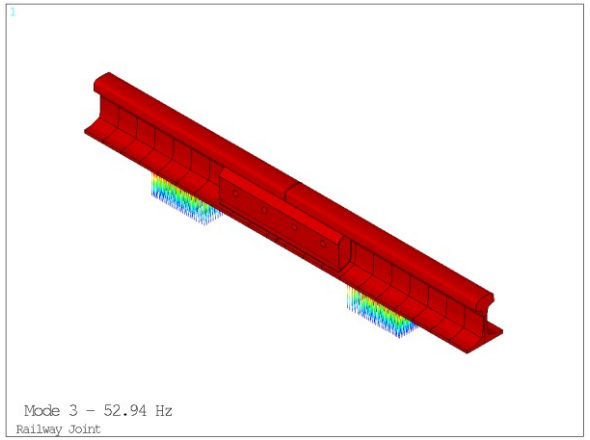


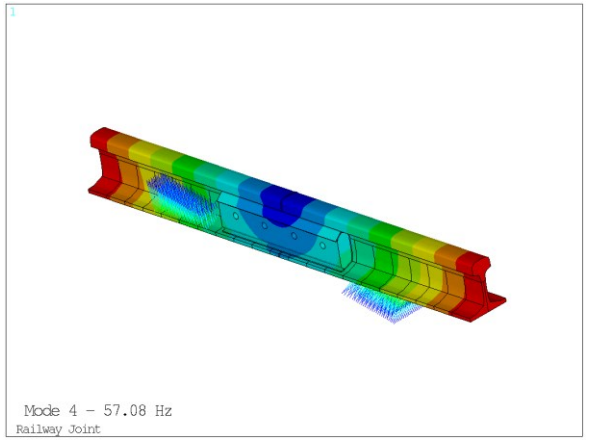
Mode	f [Hz]	x		y		z		ROT_x		ROT_y		ROT_z	
		Γ_x	$\sum m_x$	Γ_y	$\sum m_y$	Γ_z	$\sum m_z$	$\Gamma_{\theta x}$	$\sum m_{\theta x}$	$\Gamma_{\theta y}$	$\sum m_{\theta y}$	$\Gamma_{\theta z}$	$\sum m_{\theta z}$
1	24.17	0.00	0.00	0.00	0.00	**9.17**	0.62	**1.18**	1.00	0.00	0.00	0.00	0.00
2	42.18	**11.22**	0.93	0.00	0.00	0.00	0.62	0.00	1.00	0.00	0.00	**-2.10**	0.19
3	52.93	0.00	0.93	**11.61**	1.00	0.00	0.62	0.00	1.00	0.00	0.00	0.00	0.19
4	57.08	**3.00**	1.00	0.00	1.00	0.00	0.62	0.00	1.00	0.00	0.00	**4.29**	1.00
5	127.54	0.00	1.00	0.00	1.00	0.00	0.62	0.00	1.00	**4.60**	0.98	0.00	1.00
6	202.48	0.00	1.00	0.00	1.00	**-4.08**	0.75	**0.01**	1.00	0.00	0.98	0.00	1.00
7	220.15	0.00	1.00	0.00	1.00	**5.83**	1.00	**-0.01**	1.00	0.00	0.98	0.00	1.00
8	333.55	0.00	1.00	**0.08**	1.00	0.00	1.00	0.00	1.00	0.00	0.98	0.00	1.00
9	417.02	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	**-0.65**	1.00	0.00	1.00
10	482.43	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	**-0.24**	1.00	0.00	1.00
11	540.38	0.00	1.00	0.00	1.00	**0.06**	1.00	0.00	1.00	0.00	1.00	0.00	1.00
12	937.65	0.00	1.00	0.00	1.00	**-0.21**	1.00	0.00	1.00	0.00	1.00	0.00	1.00
13	958.17	-0.01	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	**0.01**	1.00

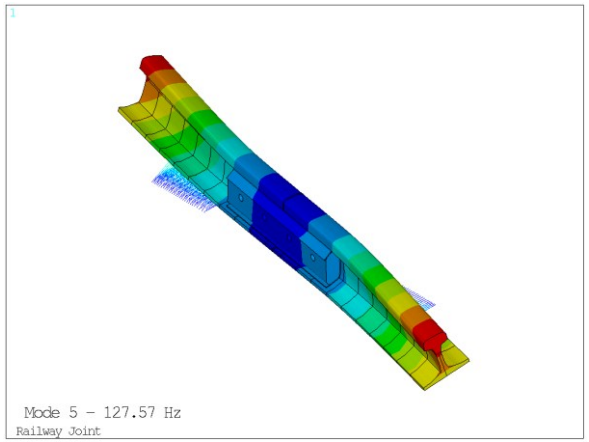
Mode shapes

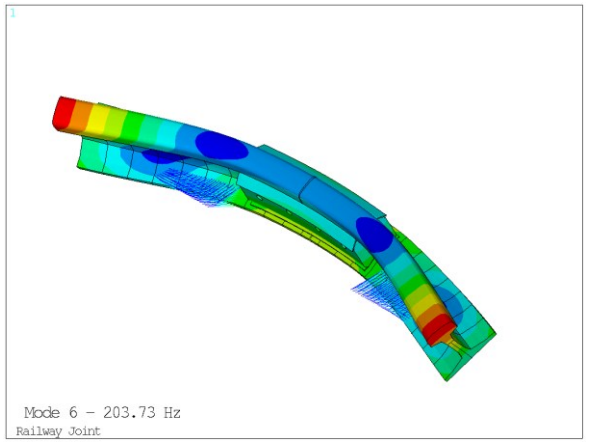

Mode 1 – 24.16 Hz
Railway Joint

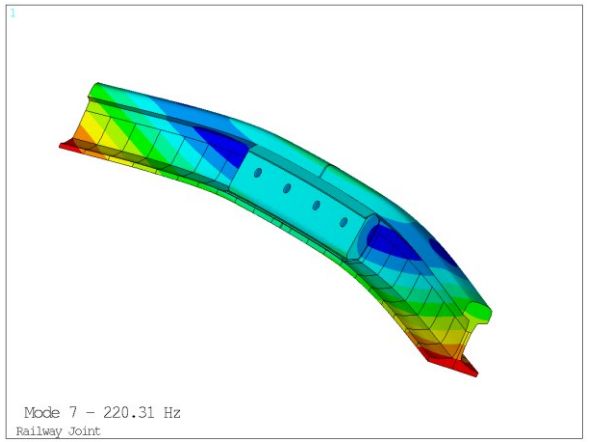

Mode 2 – 42.17 Hz
Railway Joint

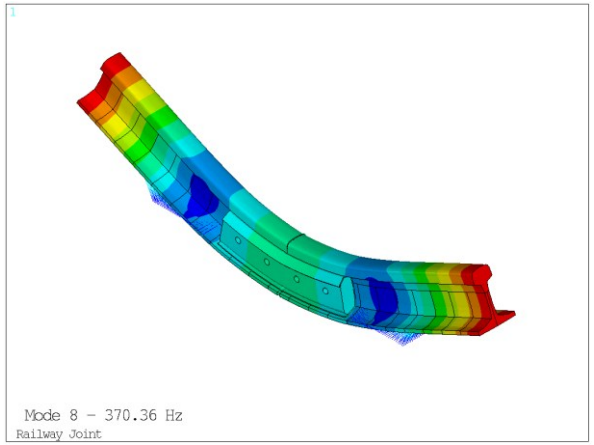

Mode 3 – 52.94 Hz
Railway Joint


Mode 4 – 57.08 Hz
Railway Joint

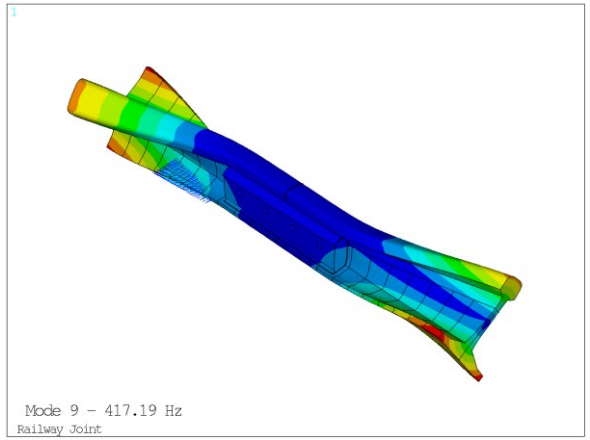

Mode 5 – 127.57 Hz
Railway Joint

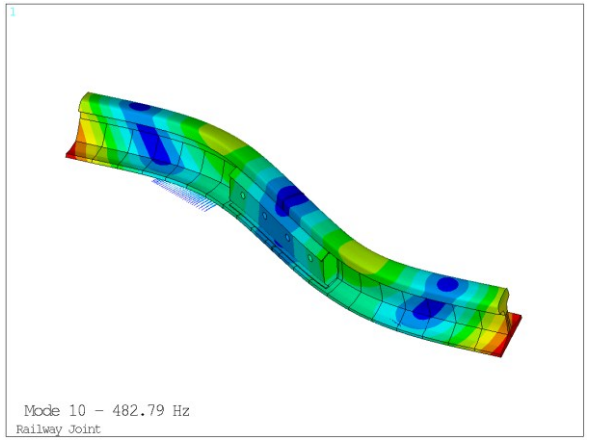

Mode 6 – 203.73 Hz
Railway Joint

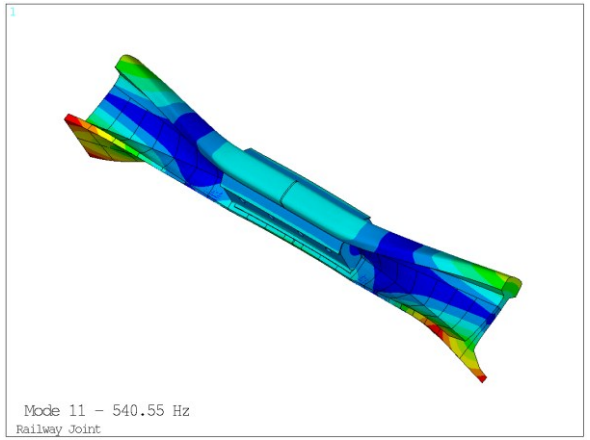

Mode 7 – 220.31 Hz
Railway Joint

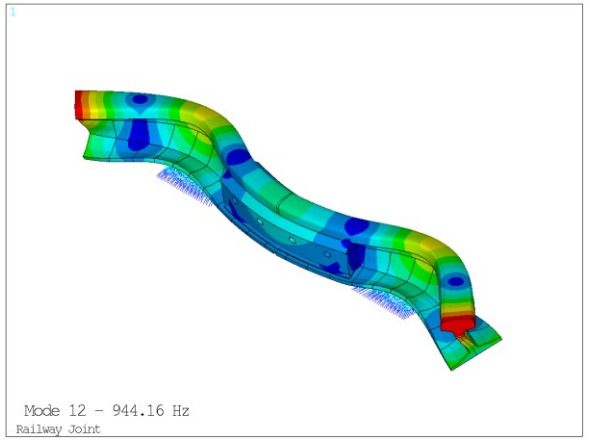

Mode 8 – 370.36 Hz
Railway Joint

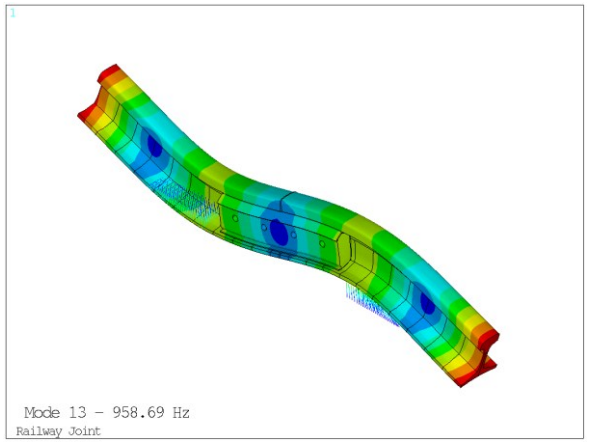
Mode shapes

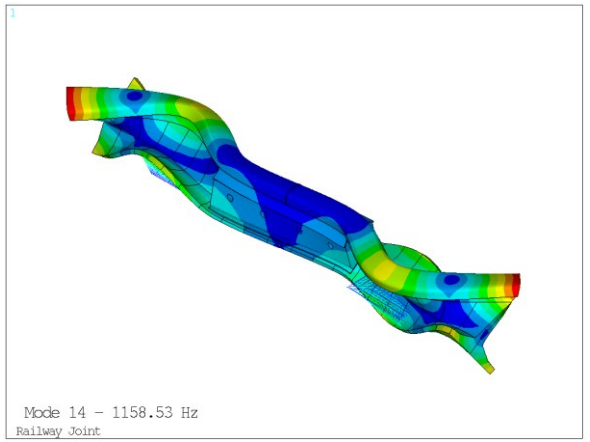

Mode 9 – 417.19 Hz
Railway Joint

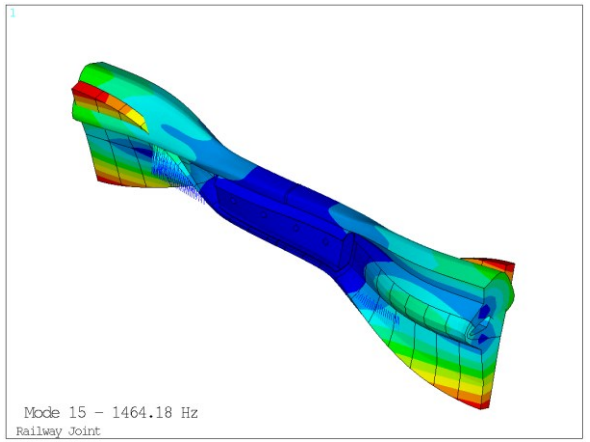

Mode 10 – 482.79 Hz
Railway Joint

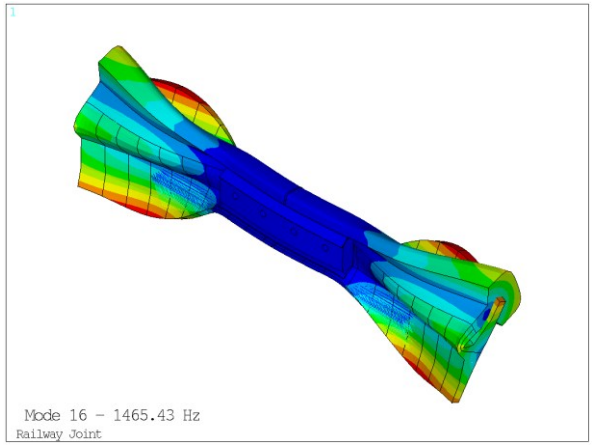

Mode 11 – 540.55 Hz
Railway Joint


Mode 12 – 944.16 Hz
Railway Joint


Mode 13 – 958.69 Hz
Railway Joint


Mode 14 – 1158.53 Hz
Railway Joint


Mode 15 – 1464.18 Hz
Railway Joint

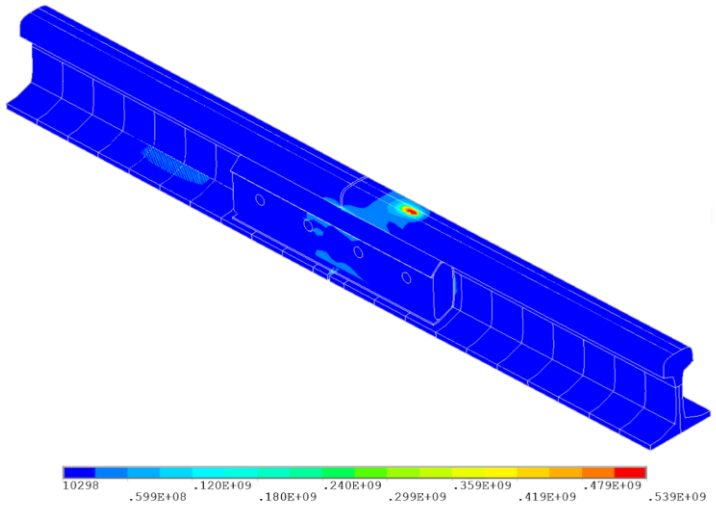

Mode 16 – 1465.43 Hz
Railway Joint

Data analysis

Raw data extraction
- Load step
- Extract x component of **EPEL** in global coordinate system
- Save to **.csv**

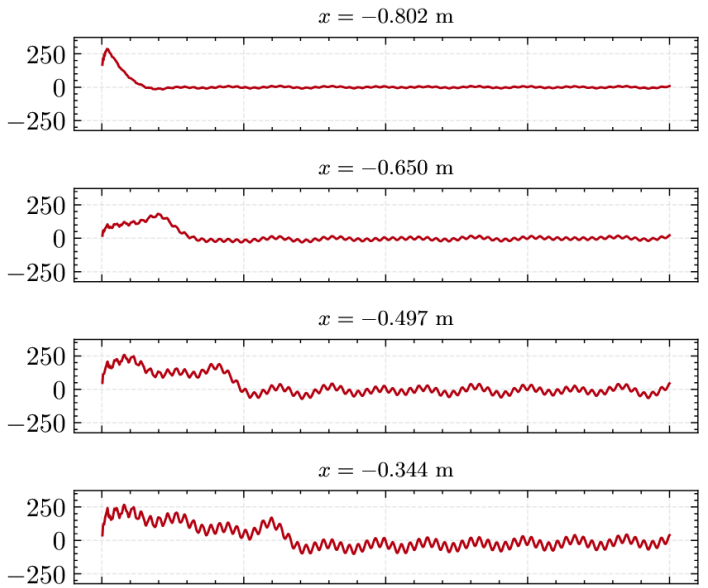
Cubic spline interpolation
- Data interpolation in Python
- Wave reconstruction
- **Physical justification**: a cubic spline minimizes the integral of the second spatial derivative

Spectral density analysis
- Using **Welch's method** we can **compute the Power spectral density**
- Both **per probe spectra** and **average spectrum**



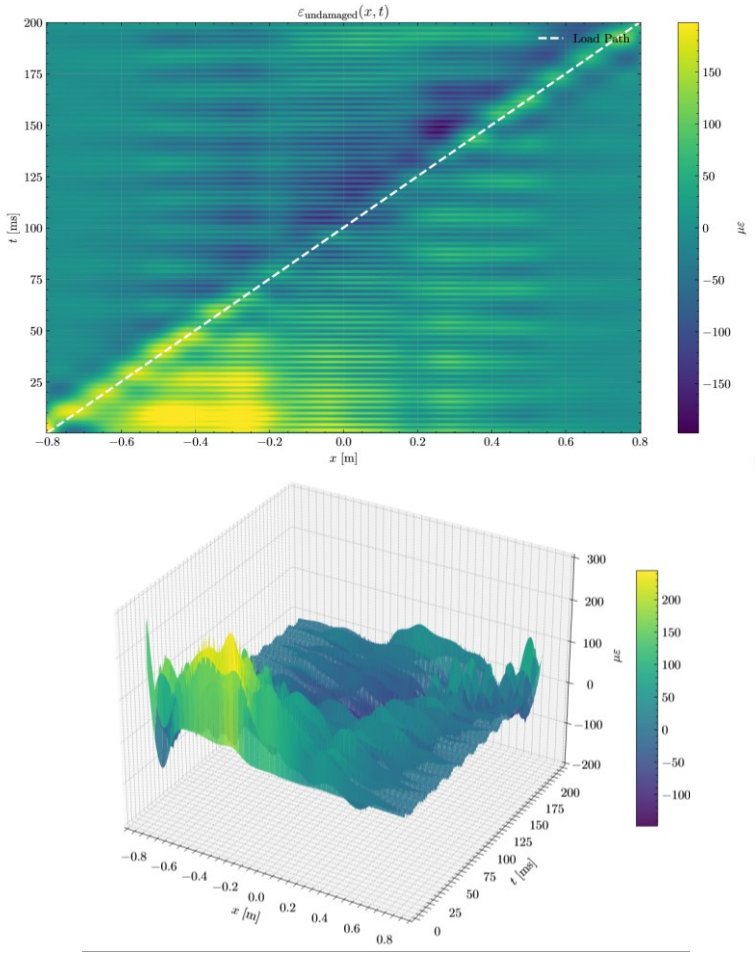
$$\min \int_0^L [S''(x)]^2 dx$$

$$U = \frac{1}{2} \int_0^L EI \left(\frac{\partial^2 v}{\partial x^2} \right)^2 dx$$

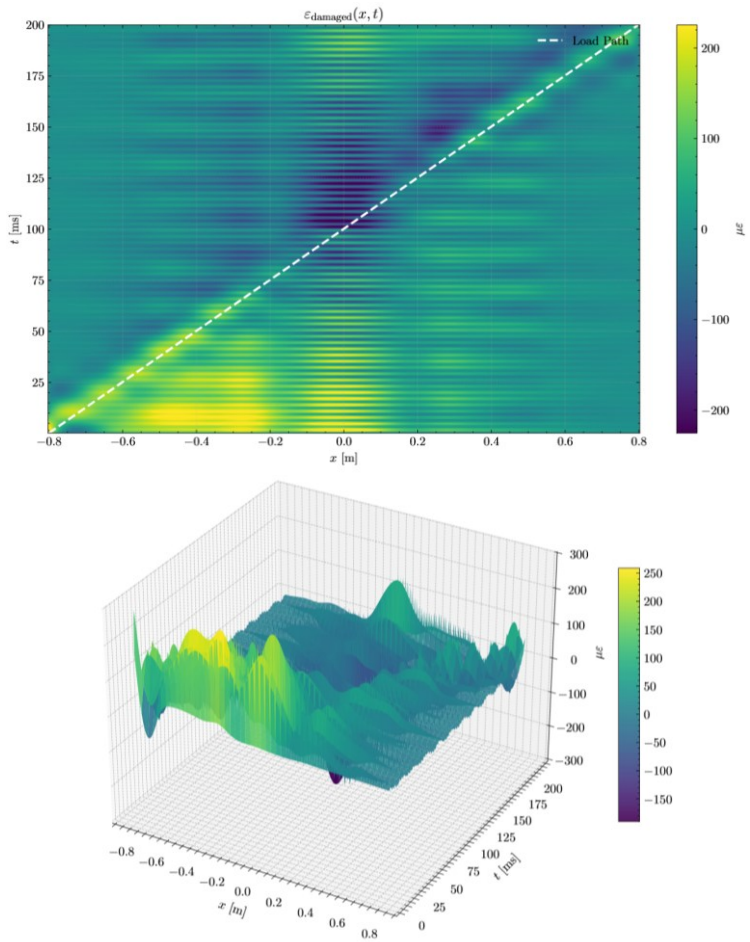


Wave reconstruction

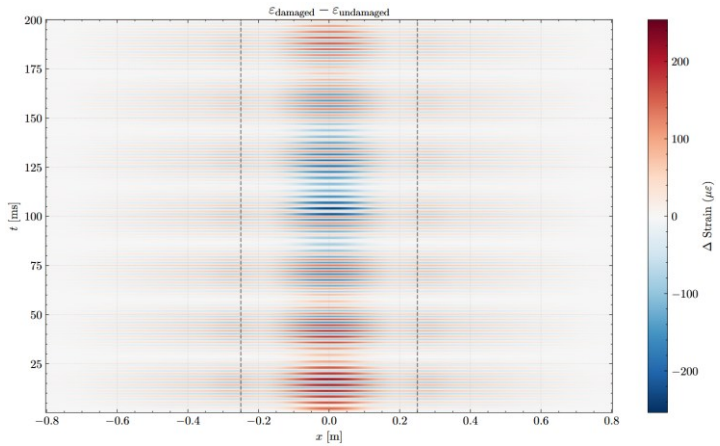
Undamaged



Damaged

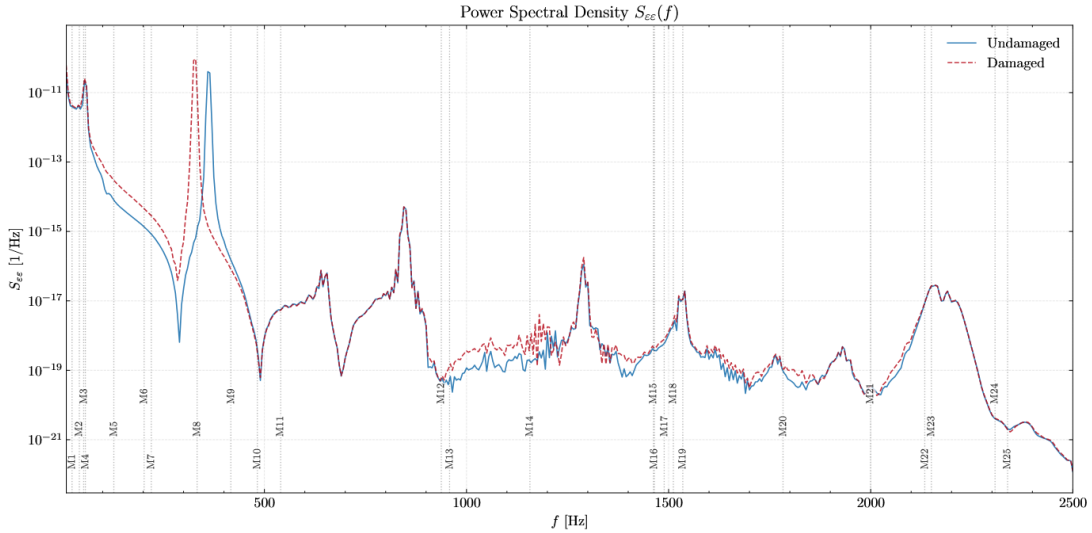


Difference

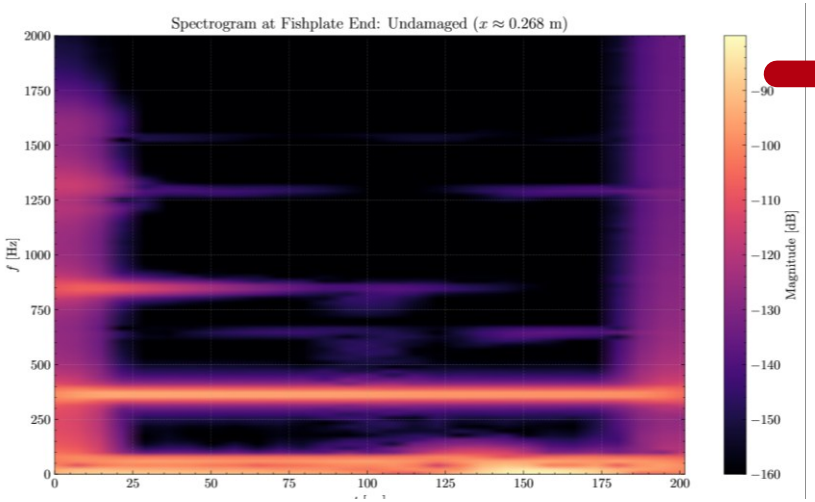


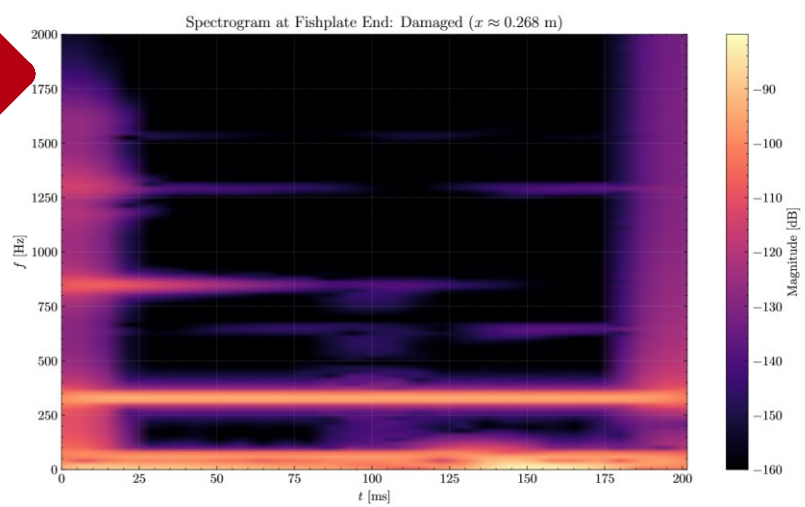
- **Localized** strain variation (**±200με**)
- Difference **strictly confined** within the **fishplate** boundaries (**±0.25m**)
- The damaged joint acts as a compliant connection undergoing **oscillatory opening and closing cycles**

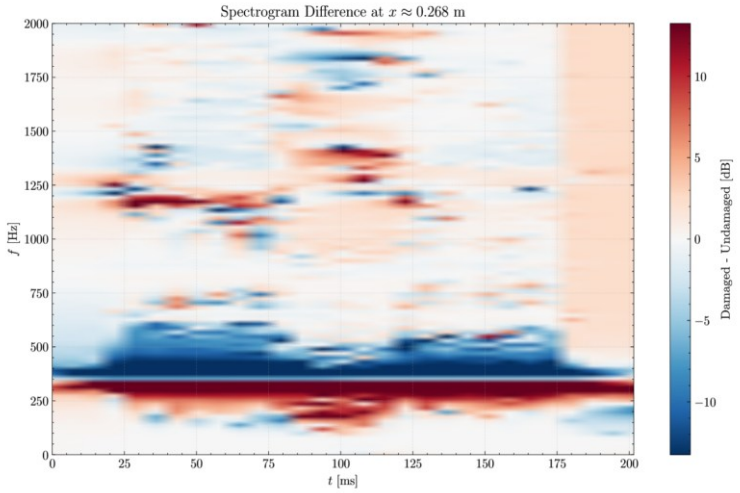
Spectral analysis



- Low frequency semi-rigid modes **unaffected by the damage**
- Manifest frequency shift from **360Hz** to **330Hz** from decreased local stiffness







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Mesh size verification

Target: at least 4-6 elements per wavelength in order to resolve the wave

Benchmark frequency: Nyquist frequency limit $f_n = 2500$ Hz, provides a **conservative** safety margin as frequencies of interest are an order of magnitude lower

Wave propagation equations:

- **Longitudinal**

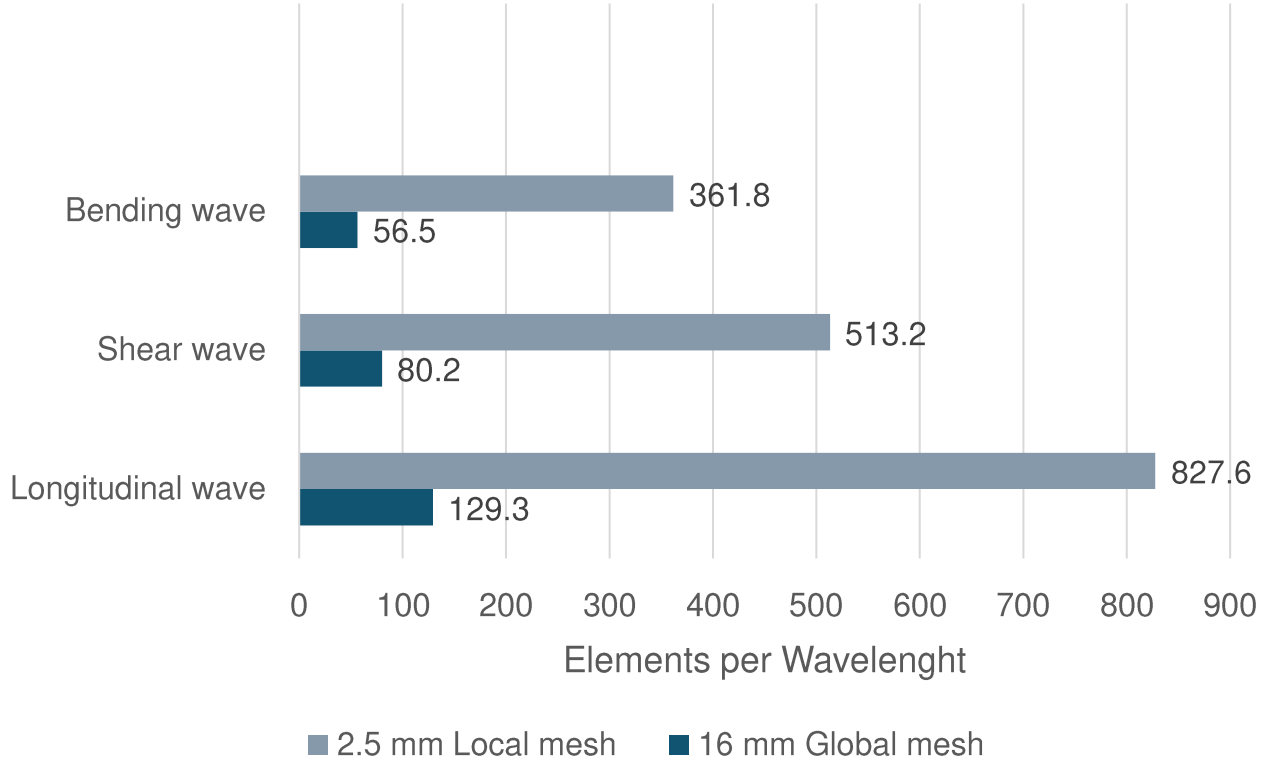
$$\lambda_L = \frac{C_L}{f_N} = \frac{\sqrt{E/\rho}}{f_n} \approx 2069 \text{ mm}$$

- **Shear**

$$\lambda_S = \frac{C_S}{f_N} = \frac{\sqrt{G/\rho}}{f_n} \approx 3206 \text{ mm}$$

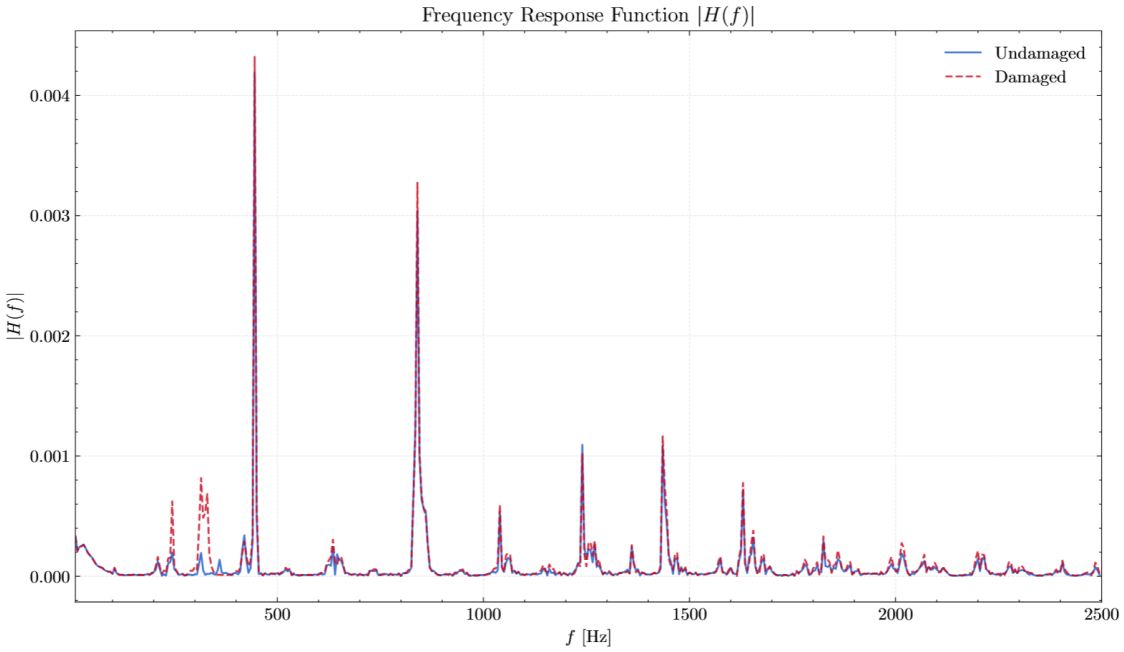
- **Bending**

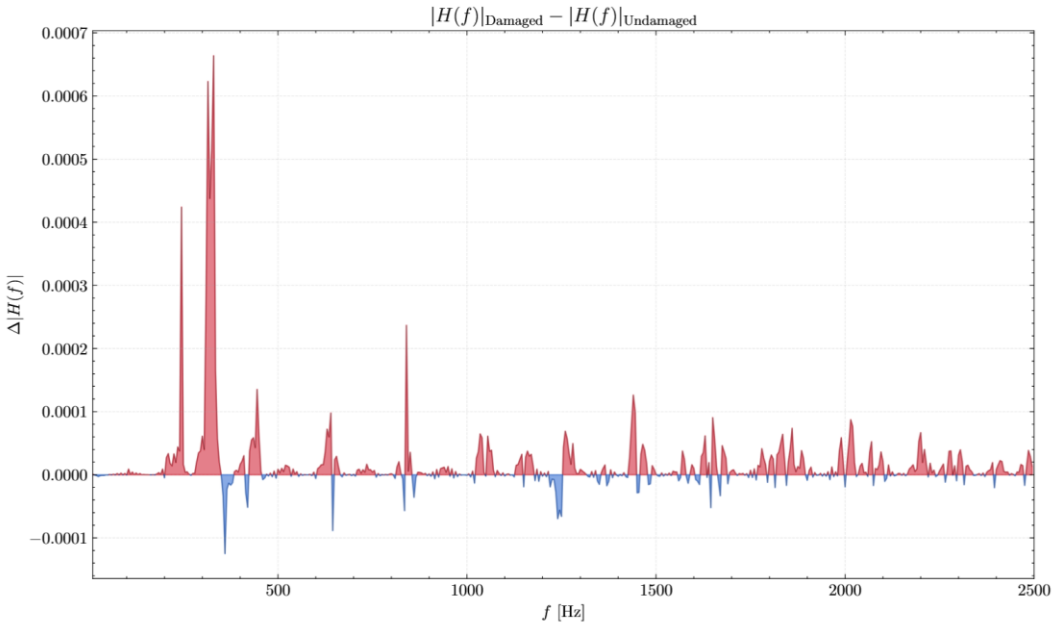
$$\lambda_B = \sqrt{\frac{2\pi}{f_N}} \left(\frac{EI}{\rho A}\right)^{1/4} \approx 904.6 \text{ mm}$$



Even the worst-case scenario (16mm element on bending waves) **exceeds by an order of magnitude** the minimum number of elements required

Spectral analysis





Frequency response function

- Averaged probe spectrum divided by the 1D hertzian load spectrum
- Removes external force influence
- The frequency shift is still **clearly present**

$$H_i(f) = \frac{\mathcal{F}\{\varepsilon_i(t)\}}{\mathcal{F}\{f_i(t)\}}$$

$$H(f) = \frac{1}{N_{\text{probes}}} \sum_{i=1}^{N_{\text{probes}}} |H_i(f)|$$

Damage detection strategy

The danger:

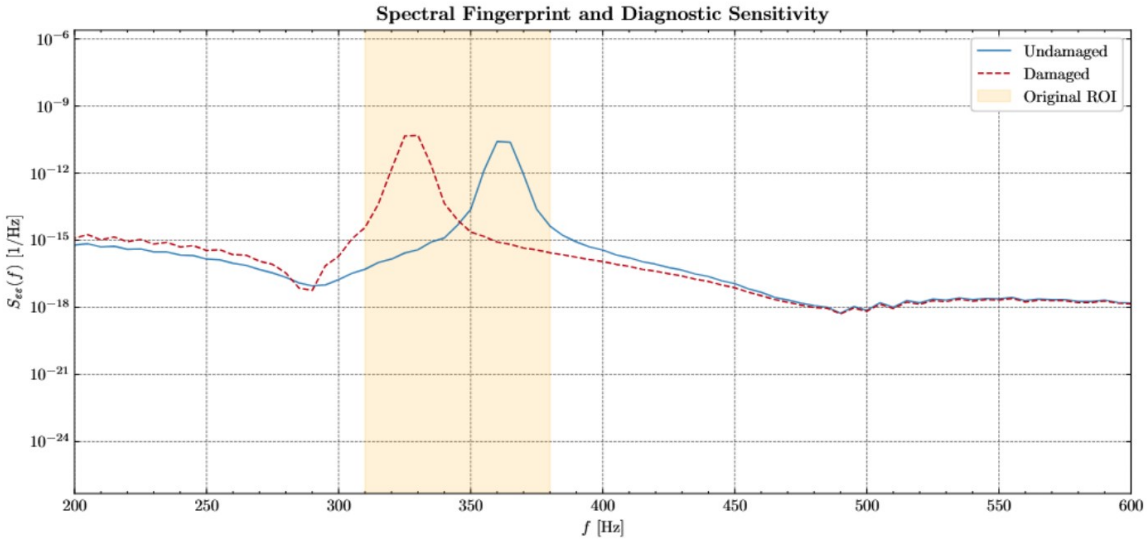
- Broken end-posts cause electrical short circuits leading to **False Clear** (collision risk) or **False Occupancy** (rail network blockage) signals

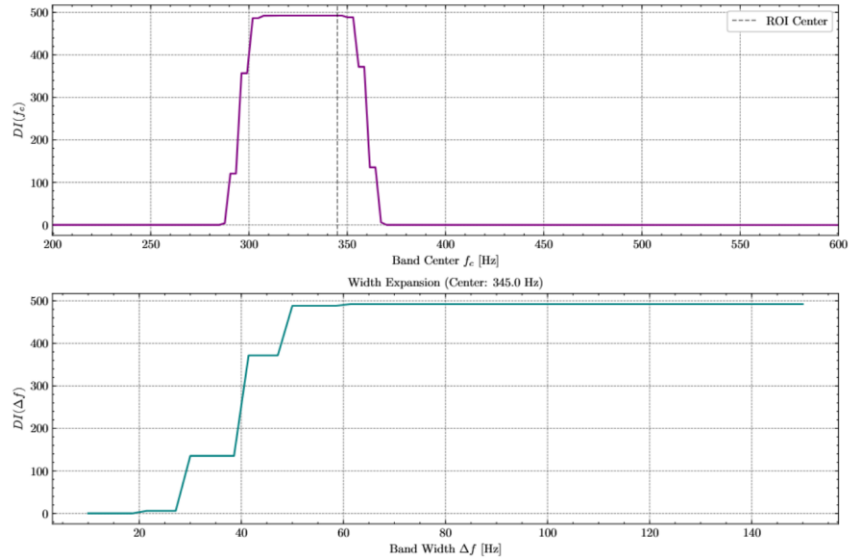
Diagnostic Index:

- Frequency band: $[f_1, f_2] = [310{,}380] Hz$
- $DI \gg 1$ **Failure of the end-post**

$$DI = \int_{f_1}^{f_2} \frac{S_{\varepsilon\varepsilon,\text{measured}}(f)}{S_{\varepsilon\varepsilon_{\text{undamaged}}}(f)} df$$

- **Sensitivity analysis** to band centre and bandwidth
- For this simulation the **DI** returns a value of around **500**, **correctly detecting the complete failure of the end-post**.
- A partial failure would **return lower values**







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